Possible solution of the Coriolis attenuation problem

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Abstract

The most consistently useful simple model for the study of odd deformed nuclei, the particle-rotor model (strong coupling limit of the core-particle coupling model) has nevertheless been beset by a long-standing problem: It is necessary in many cases to introduce an *ad hoc* parameter that reduces the size of the Coriolis interaction coupling the collective and single-particle motions. Of the numerous suggestions put forward for the origin of this supplementary interaction, none of those actually tested by calculations has been accepted as the solution of the problem. In this paper we seek a solution of the difficulty within the framework of a general formalism that starts from the spherical shell model and is capable of treating an arbitrary linear combination of multipole and pairing forces. With the restriction of the interaction to the familiar sum of a quadrupole multipole force and a monopole pairing force, we have previously studied a semi-microscopic version of the formalism whose framework is nevertheless more comprehensive than any previously applied to the problem. We obtained solutions for low-lying bands of several strongly deformed odd rare earth nuclei and found good agreement with experiment, except for an exaggerated staggering of levels for $K = \frac{1}{2}$ bands, which can be understood as a manifestation of the Coriolis attenuation problem. We argue that within the formalism utilized, the only way to improve the physics is to add interactions to the model Hamiltonian. We verify that by adding a magnetic dipole interaction of essentially fixed strength, we can fit the $K = \frac{1}{2}$ bands without destroying the agreement with other bands. In addition we show that our solution also fits ^{163}Er, a classic test case of Coriolis attenuation that we had not previously studied.

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I. INTRODUCTION

In this paper we propose a solution to a long-standing conundrum in the theoretical description of the properties of low-lying bands of deformed odd nuclei within the framework of the particle-rotor model [1]. Generally speaking, the Hamiltonian of this model has the form

$$H = H_{\rm rotor} + H_{\rm particle}. \tag{1}$$

Here the first term is a phenomenological operator describing the included bands of a given even (core) nucleus, assumed to be a rotor. In the simplest case, it describes only the ground band of an axially symmetric system. The second term contains the single-particle energies of and the residual interactions among the particles that move outside the rotor and are assumed to be kinematically independent of the constituents of this core. When we wish to describe odd nuclei that are adjacent to the given core, the simplest assumption is that it suffices to consider a single extra particle, or if we include pairing interactions, quasiparticle, outside the core, described in the intrinsic system of the rotor by a set of axially symmetric solutions of the Nilsson-BCS formalism.

An important talking point in favor of this extreme model is that it contains, in so far as its predictions of energy levels is concerned, essentially no free parameters, since the quasiparticle properties can be determined by fitting bandheads alone, and the core Hamiltonian depends to a good approximation only on the observed moment of inertia of its ground-state band. When the angular momentum of the core is replaced, according to angular momentum conservation, by the difference between the total angular momentum $\mathbf{J}$ and the particle angular momentum $\mathbf{j}$, the interaction between particle and core becomes (omitting BCS occupation factors)

$$\frac{\mathbf{j}\cdot\mathbf{J}}{2\mathcal{I}}, \tag{2}$$

where $\mathcal{I}$ is the moment of inertia of the core.

For many applications, the model as described above does not predict the observed band structure [2,3] unless one multiplies the Coriolis interaction by a factor $\xi \cong .6 - .8$, thus giving rise to the name Coriolis Attenuation. This means that the model as described above is too rudimentary, and that there is missing physics. There have been numerous suggestions concerning the source of this missing physics. We shall not review these suggestions in the form that they are described in the literature [2,3] for at least two reasons. The first is that all those that have actually been tested by calculations fall short in some respect. The second is that they are all stated within the framework of some extended version of the particle-rotor model. But this model has among its flaws the assumption of the kinematic independence of the particle and rotor variables. This certainly violates the Pauli principle, the more so if one wishes to allow the particle to have access to several major shells (as in the calculations we report in the present work).

Instead we shall approach the subject from the point of view of a theoretical framework that is formally exact and contains the particle-rotor model as a well-defined limit. We claim that for a sufficiently simple Hamiltonian we can, within this framework, compute accurate solutions for the low lying bands of well-deformed odd nuclei. Therefore with high

probability a failure of the theory is to be ascribed to a missing term in the Hamiltonian. For example, one of the classes of proposed explanation within the particle-rotor framework, that we need more than one particle outside the core, is ruled out in our case by the fact that the exact formalism never involves more than one particle outside a band of the core, the point being that the inclusion of core excitation is taken care of by the inclusion of excited bands of the core. Another explanation of the standard list is that the model in its simplest form omits or treats incorrectly the so-called recoil term that arises from the same transformation that yields the Coriolis coupling; the distinction between the two terms is a consequence of the use of an intrinsic frame in the implementation of the model. But our calculations are carried in the so-called laboratory system, in a way that automatically includes the effects of the recoil term. The last important class of suggestions and the only one that is cogent from the point of view of the theory we utilize is that of a missing interaction. Here, the structure of our equations suggests an essentially unique choice, at least as far as multipolarity is concerned. Further discussion of this point, the major one made in this paper, is continued below, after some remarks about the formalism.

The theory of collective motion for odd nuclei that we shall apply is one that we have recently resuscitated and extended, and that we have rechristened the Kerman-Klein-Dönau-Frauendorf (KKDF) model [4–8]. This model was introduced by Dönau and Frauendorf [9–12,2], whose work was in turn stimulated by an application [13] of the theory of collective motion developed by Kerman and Klein [14–17]. This theory starts from the spherical shell model with residual two-body interactions. A formally exact set of equations is derived that *resembles* the equations of a particle-core coupling model, but requires even after a suitable restriction on the size of the configuration space, the solution of a formidable non-linear problem. In the KKDF model the problem is linearized by choosing the matrix elements of multipole and pairing operators, that arise from the interaction, either phenomenologically or from experiment. If this can be done for a sufficiently extensive model space, we can be reasonably confident that the corresponding equations of the non-linear theory have been solved accurately (although we have no proof of this statement). The KKDF program can be carried out at the present time only if the interaction chosen is sufficiently simple that in fact the "core" matrix elements that enter can be determined at least in part from experiment.

In previous work we have used a conventional Hamiltonian in which the interactions were confined to monopole pairing and quadrupole multipole interactions. Among the applications made, the simplest have been to the low lying bands of several well-deformed odd rare earth nuclei. To a good approximation the only free parameter was the strength of the quadrupole interaction. With the important exception to which this paper is devoted, we found good agreement with the energies and electromagnetic moments (where measured) of all the lowest lying bands, provided the calculations were carried out with three major shells for the single-particle space and with the inclusion of excited bands of the core (including but not confined to beta and gamma bands). The inclusion of the excited core bands was necessary to account for odd bands near the top of the energy region considered, approximately 1 Mev.

The only discrepancy found was that our calculations predicted, uniformly, an exaggerated staggering of levels for $K = \frac{1}{2}$ bands compared to experiment. In a particle-rotor model, it is easy to see from standard formulas [1] that this discrepancy can be erased by introducing the Coriolis attenuation factor. As we have argued above, the only fundamental way

that is open to us to improve our calculations is to add an interaction to the Hamiltonian. Such an interaction should effectively contribute to our equations a Coriolis-like coupling that is dynamically independent of the standard one (the latter hidden in our formalism because we calculate in the laboratory system). Once we have reached this point in our thinking, it is almost obvious that the choice should be a magnetic dipole interaction, also satisfying our requirement that the core matrix elements can be measured. In fact, from the well-known properties of vector operators it follows that contributions diagonal in both core angular momentum and particle angular momentum of any dipole interaction are exactly equivalent to a Coriolis interaction. However, the magnetic dipole interaction can also have off-diagonal elements and therefore the equivalence is only approximate.

It may be noted that there is even a simpler way to obtain an additional Coriolis coupling than the one we have chosen, and that is to add to the Hamiltonian a term proportional to the square of the angular momentum. We shall verify that such an interaction can also be used to fit the data. Such a Coriolis coupling term was seen to arise, for example, in a paper [18] showing that a version of the particle-rotor model can be derived from the Fermion Dynamical Symmetry Model (FDSM). This is not surprising, since the standard Hamiltonian of FDSM indeed contains a term proportional to the square of the angular momentum operator of that model. This observation was not followed up.

It is also of interest to remark that in a recent analysis [19] which sought to identify the most important multipole components of a shell-model interaction, it was found that magnetic dipole interactions were present. The result was represented as a spin-spin interaction and therefore different in detail from either of the possibilities examined in this work. Nevertheless this analysis further buttresses our viewpoint.

Finally we outline the contents of the body of work that follows. In Sec. II, we briefly review the basic elements of our theory and incorporate the changes necessary to include the new interaction. In Sec. III, we specify how the parameters of the magnetic dipole interactions were chosen. The results of our calculations are presented in Sec. IV, together with a brief description of how the analysis was carried out, where it was deemed necessary to supplement discussions in our earlier papers. In Sec. V we compare some results found for the magnetic dipole interaction with a corresponding analysis in which we simply add to the Hamiltonian a term proportional to the square of the angular momentum. A brief discussion of our results is given in Sec. VI.

II. THEORY

For the sake of completeness, we start with a brief review of the basic elements of our method. We base our considerations on a shell-model Hamiltonian of the form,

$$\begin{aligned} H = \sum_a h_a a^\dagger_\alpha a_\alpha &+ \frac{1}{4}\sum_{abcd}\sum_{LM_L} F_{acdb}(L)\, B^\dagger_{LM_L}(a,c) B_{LM_L}(d,b) \\ &+ \frac{1}{4}\sum_{abcd}\sum_{LM_L} G_{abcd}(L)\, A^\dagger_{LM_L}(a,b) A_{LM_L}(c,d). \end{aligned} \tag{3}$$

Here h_a ($\alpha = (j_a, m_a)$ and $a = j_a$) are the spherical single particle energies, B_{LM} is the elementary particle-hole multipole operator,

$$B^{\dagger}_{LM_L}(a,b) \equiv \sum_{m_a m_b} s_\beta \, C^{LM_L}_{\alpha\bar{\beta}} \; a^{\dagger}_{\alpha} a_{\beta} = \left[a^{\dagger}_{a} \times \tilde{a}_{b}\right]^{L}_{M_L}, \tag{4}$$

and A_{LM} is the elementary particle-particle multipole operator

$$A^{\dagger}_{LM_L}(a,b) \equiv \sum_{m_a m_b} C^{LM_L}_{\alpha\beta} \; a^{\dagger}_{\alpha} a^{\dagger}_{\beta} = \left[a^{\dagger}_{a} \times a^{\dagger}_{b}\right]^{L}_{M_L}, \tag{5}$$

where $C^{LM}_{\alpha\beta}$ are the Clebsch-Gordan coefficients and $s_\alpha = (-)^{j_a - m_a}$. The coefficients F are the particle-hole matrix elements

$$F_{acdb}(L) \equiv \sum_{m's} s_\gamma s_\beta \, C^{LM_L}_{\alpha\bar{\gamma}} \, C^{LM_L}_{\delta\bar{\beta}} \; V_{\alpha\beta\gamma\delta}, \tag{6}$$

and G the particle-particle matrix elements

$$G_{abcd}(L) \equiv \sum_{m's} C^{LM_L}_{\alpha\beta} \, C^{LM_L}_{\gamma\delta} \, V_{\alpha\beta\gamma\delta}. \tag{7}$$

The task is to obtain equations for the states and energies of the odd nucleus assuming that the properties of neighboring even nuclei are known. The states of the odd nucleus (particle number N) are designated as $\mid J\mu\nu\rangle$ where ν denotes all quantum numbers beside the angular momentum J and its projection μ. The eigenstates and eigenvalues of the neighboring even nuclei with particle numbers $(N \pm 1)$ are $|IM\,n\,(N \pm 1)\rangle$ and $E^{N\pm1}_{In}$, respectively, where n plays the same role for even nuclei as ν does for the odd nuclei. The equations of motion (EOM) are obtained by forming commutators between the Hamiltonian and single fermion operators, leading to

$$\begin{aligned}[a_\alpha, H] = h_a a_a &+ \frac{1}{4}\sum_{bd\gamma}\sum_{LM} C^{LM}_{\alpha\gamma} G_{acbd}(L) a^{\dagger}_{\gamma} A_{LM}(c,d) \\ &+ \frac{1}{4}\sum_{bd\gamma}\sum_{LM} s_\gamma C^{LM}_{\alpha\bar{\gamma}} F_{acdb}(L) a_\gamma B_{LM}(d,b),\end{aligned} \tag{8}$$

together with its hermitian conjugate.

The matrix elements of these equations provide expressions that determine the single-particle coefficients of fractional parentage (cfp),

$$V_{J\mu\nu}(\alpha; IMn) = \langle J\mu\nu|a_\alpha|IMn(A+1)\rangle, \tag{9}$$
$$U_{J\mu\nu}(\alpha; IMn) = \langle J\mu\nu|a^{\dagger}_{\bar{\alpha}}|IMn(A-1)\rangle. \tag{10}$$

In terms of a convenient and physically meaningful set of energy differences and sets of multipole fields and pairing fields defined below, we obtain generalized matrix equations of the Hartree-Bogoliubov form

$$\begin{aligned}\mathcal{E}_{J\nu} V_{J\mu\nu}(\alpha; IMn) = &(\epsilon + \omega^{(A+1)} + \Gamma^{(A+1)})_{\alpha IMn,\gamma I'M'n'} V_{J\mu\nu}(\gamma; I'M'n') \\ &+ \Delta_{\alpha IMn,\gamma I'M'n'} U_{J\mu\nu}(\gamma; I'M'n'),\end{aligned} \tag{11}$$
$$\begin{aligned}\mathcal{E}_{J\nu} U_{J\mu\nu}(\alpha; IMn) = &(-\epsilon + \omega^{(A-1)} - \Gamma^{(A-1)\dagger})_{\bar{\alpha} IMn,\bar{\gamma} I'M'n'} U_{J\mu\nu}(\gamma; I'M'n') \\ &+ \Delta^{\dagger}_{\bar{\alpha} IMn,\bar{\gamma} I'M'n'} V_{J\mu\nu}(\gamma; I'M'n'),\end{aligned} \tag{12}$$

where

$$\mathcal{E}_{J\nu} = -E_{J\nu} + \frac{1}{2}(E_0^{(A+1)} + E_0^{(A-1)}), \tag{13}$$

$$\epsilon_{\alpha IMn,\gamma I'M'n'} = \delta_{\alpha\gamma}\delta_{II'}\delta_{MM'}\delta_{nn'}(h_a - \lambda_A), \tag{14}$$

$$\lambda_A = \frac{1}{2}(E_0^{(A+1)} - E_0^{(A-1)}), \tag{15}$$

$$\omega^{(A\pm1)}_{\alpha INn,\gamma I'M'n'} = \delta_{\alpha\gamma}\delta_{II'}\delta_{MM'}\delta_{nn'}(E^{(A\pm1)}_{In} - E_0^{(A\pm1)}), \tag{16}$$

$$\begin{aligned}\Gamma^{(A\pm1)}_{\alpha IMn,\gamma I'M'n'} &= \frac{1}{2}\sum_{LM_L}\sum_{bd} s_\gamma C^{LM_L}_{\alpha\bar{\gamma}}\, F_{acdb}(L),\\ &\quad \langle I'M'n'(A\pm1)|B_{LM_L}(db)|IMn(A\pm1)\rangle\end{aligned} \tag{17}$$

$$\begin{aligned}\Delta_{\alpha IMn,\gamma I'M'n'} &= \frac{1}{2}\sum_{LM_L}\sum_{bd} C^{LM_L}_{\alpha\bar{\gamma}}\, G_{acdb}(L)\\ &\quad \langle I'M'n'(A-1)|A_{LM_L}(db)|IMn(A+1)\rangle.\end{aligned} \tag{18}$$

Here $\mathcal{E}$ is the eigenvalue, the energy of the state of an odd nucleus relative to the average ground state energy of the neighboring even ones. The remaining definitions refer to the elements of the effective Hamiltonian matrix: ϵ are the single particle energies measured relative to the chemical potential, λ the chemical potential, the elements of ω are the excitation energies in the even nuclei, and Γ and Δ are the multipole and pair fields, respectively.

The solutions also require a normalization condition that is derived from matrix elements of the fundamental anticommutation relations,

$$\frac{1}{\Omega}\sum_{\alpha IMn}\left[|U^\nu_{J\mu}(\alpha IM;n)|^2 + |V^\nu_{J\mu}(\alpha IM;n)|^2\right] = 1, \tag{19}$$

where

$$\Omega = \sum_{j_a}(2j_a + 1). \tag{20}$$

Since we shall use only a restricted set of interactions, specializing to the case where $L = 1, 2$ multipoles and the $L = 0$ pairing are included, the multipole and pairing fields defined above take the form

$$\begin{aligned}\Gamma^{(A\pm1)}_{\alpha IMn,\gamma I'M'n'} &= \frac{1}{2}\sum_{M_1}\sum_{bd} s_\gamma\; C^{1M_1}_{\alpha\bar{\gamma}} F_{acdb}(1),\\ &\quad \langle I'M'n'(A\pm1)|B_{1M_1}(db)|IMn(A\pm1)\rangle\\ &\quad +\frac{1}{2}\sum_{M_2}\sum_{bd} s_\gamma\; C^{2M_2}_{\alpha\bar{\gamma}} F_{acdb}(2),\\ &\quad \langle I'M'n'(A\pm1)|B_{2M_L}(db)|IMn(A\pm1)\rangle\end{aligned} \tag{21}$$

$$\begin{aligned}\Delta_{\alpha IMn,\gamma I'M'n'} &= \frac{1}{2}\sum_{bd} C^{00}_{\alpha\bar{\gamma}} G_{acdb}(0)\\ &\quad \langle I'M'n'(A-1)|A_{00}(db)|IMn(A+1)\rangle.\end{aligned} \tag{22}$$

The further specialization to the magnetic dipole, the mass quadrupole, and the standard monopole pairing interactions, is defined by the equations

$$F_{acdb}(1) = -\kappa_1\, M_{ab}(1)\, M_{dc}(1), \tag{23}$$
$$F_{acdb}(2) = -\kappa_2\, F_{ab}(2)\, F_{dc}(2), \tag{24}$$
$$G_{acdb}(0) = -g_0\, G_{ab}(0)\, G_{dc}(0), \tag{25}$$

together with the identification of the sums

$$\mathcal{M}_\mu = \sum_{ac} M_{ac}(1) B_{1M_1}(a,c), \tag{26}$$
$$Q_\mu = \sum_{ac} F_{ac}(2) B_{2M_2}(a,c), \tag{27}$$
$$\Delta = \sum_{ac} G_{ac}(0) A_{00}(a,c), \tag{28}$$

as the electromagnetic dipole, the mass quadrupole, and the pairing gap operators, respectively.

With the definitions given above, the multipole and pairing fields take the form

$$\begin{aligned}\Gamma^{(A\pm1)}_{\alpha IMn,\gamma I'M'n'} = &-\frac{\kappa_1}{2}\sum_{M_1} s_\gamma\ C^{1M_1}_{\alpha\bar\gamma} M_{ac}(1)\\ &\times\langle I'M'n'(A\pm1)|\mathcal{M}_{M_1}(db)|IMn(A\pm1)\rangle\\ &-\frac{\kappa_2}{2}\sum_{M_2} s_\gamma\ C^{2M_2}_{\alpha\bar\gamma} F_{ac}(2),\\ &\times\langle I'M'n'(A\pm1)|Q_{M_2}(db)|IMn(A\pm1)\rangle, \end{aligned}\tag{29}$$
$$\begin{aligned}\Delta_{\alpha IMn,\gamma I'M'n'} = &-\frac{g_0}{2} C^{0}_{\alpha\bar\gamma} G_{ac}(0)\\ &\times\langle I'M'n'(A-1)|\Delta|IMn(A+1)\rangle. \end{aligned}\tag{30}$$

We apply the Wigner-Eckart theorem to obtain the equations of motion in their final form. For the multipole and pairing fields, we use the following definitions of reduced matrix elements

$$\langle I'M'n'(A\pm1)|\mathcal{M}_{M_1}|IMn(A\pm1)\rangle = \frac{(-1)^{I-M}}{\sqrt{3}}\, C^{1M_1}_{IM\,I'-M'}\ \langle I'n'(A\pm1)||\mathcal{M}||In(A\pm1)\rangle, \tag{31}$$
$$\langle I'M'n'(A\pm1)|Q_{M_2}|IMn(A\pm1)\rangle = \frac{(-1)^{I-M}}{\sqrt{5}}\, C^{2M_2}_{IM\,I'-M'}\ \langle I'n'(A\pm1)||Q||In(A\pm1)\rangle, \tag{32}$$
$$\langle I'M'n'(A\pm1)|\Delta|IMn(A\pm1)\rangle = (-1)^{I-M}\, C^{00}_{IM\,I'-M'} \langle I'n'(A\pm1)||\Delta||In(A\pm1)\rangle, \tag{33}$$

and for the cfp V and U, we employ the definitions

$$V_{J\mu\nu}(\alpha, IMK) = \frac{(-1)^{J-\mu}}{\sqrt{2j_a+1}}\, C^{\alpha}_{IM\,J-\mu}\ \mathcal{V}_{J\nu}(aIK), \tag{34}$$
$$U_{J\mu\nu}(\alpha, IMK) = \frac{(-1)^{J-\mu+j_a+m_a}}{\sqrt{2j_a+1}} C^{\alpha}_{IM\,J-\mu}\, \mathcal{U}_{J\nu}(aIK). \tag{35}$$

We thus find the equations of motion

$$
\begin{aligned}
\mathcal{E}_{J\nu}\ \mathcal{V}_{J\nu}(aIK) = (\epsilon_a + \omega_{IK})\ \mathcal{V}_{J\nu}(aIK) \\
+ \sum_{cI'K'} \Gamma^{(A+1)}_{aIK,cI'K'}\ \mathcal{V}_{J\nu}(cI'K') \\
+ \sum_{cI'K'} \Delta_{aIK,cI'K'}\ \mathcal{U}_{J\nu}(cI'K'),
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
\mathcal{E}_{J\nu}\ \mathcal{U}_{J\nu}(aIK) = (-\epsilon_a + \omega_{IK})\ \mathcal{U}_{J\nu}(aIK) \\
- \sum_{cI'K'} \Gamma^{(A-1)}_{aIK,cI'K'}\ \mathcal{U}_{J\nu}(cI'K') \\
+ \sum_{cI'K'} \Delta_{aIK,cI'K'}\ \mathcal{V}_{J\nu}(cI'K'),
\end{aligned} \tag{37}
$$

where

$$
\begin{aligned}
\Gamma^{(A\pm1)}_{aIK,cI'K'} &= -\frac{\kappa_1}{2}(-1)^{j_c+I+J} \begin{Bmatrix} j_a & j_c & 1 \\ I' & I & J \end{Bmatrix} \langle I'K'(A\pm1)||\mathcal{M}||IK(A\pm1)\rangle M_{ac}(1) \\
&- \frac{\kappa_2}{2}(-1)^{j_c+I+J} \begin{Bmatrix} j_a & j_c & 2 \\ I' & I & J \end{Bmatrix} \langle I'K'(A\pm1)||Q||IK(A\pm1)\rangle F_{ac}(2),
\end{aligned} \tag{38}
$$

$$
\Delta_{aIn,cI'n'} = -\frac{g_0}{2}\langle I'n'(A\pm1)||\Delta||In(A\pm1)\rangle. \tag{39}
$$

III. DETERMINATION OF MAGNETIC PARAMETERS

In this section we describe how to extract the magnetic parameters for the core and for the single particle motion.

A. Phenomenology of the even core

The matrix elements of the operator $\mathcal{M}$ between states of the even nuclei have to be determined. The reduced matrix elements of this operator can be expressed using the Wigner-Eckart theorem as

$$
\langle I||\mathcal{M}||I\rangle = \frac{\sqrt{2I+1}}{C^{II}_{II10}}\ \langle II|\mathcal{M}_0|II\rangle \tag{40}
$$

From the definition of the intrinsic magnetic moment μ we have

$$
\mu = \langle II|\mathcal{M}_0|II\rangle \tag{41}
$$

Consequently we have

$$
\langle I||\mathcal{M}||I\rangle = \frac{\sqrt{2I+1}}{C^{II}_{II10}}\ \mu \tag{42}
$$

$$
\tag{43}
$$

Because of the limitation on available experimental data for the intrinsic magnetic moments of the states of the neighboring even nuclei, we have chosen to represent their values by the simplest possible phenomenological model,

$$\mu = g_R(I) I \simeq g_1 I, \tag{44}$$

where the parameter g_R is the effective g factor for the rotational motion. Though this quantity can itself depend on the angular momentum, we have found that the use of a constant value, g_1, provides a reasonable, though not perfect, fit to the data and, at the same time, simplifies our study of the odd systems. For the resulting values of g_1 see Table I.

B. Matrix elements of the M1 operator for a single particle

For the matrix elements of the M1 operator between single particle states we quote the formula from [1] as

$$\begin{aligned} \langle j_a \parallel \mathcal{M} \parallel j_c \rangle &= \langle n_a l_a j_a \mid n_c l_c j_c \rangle \sqrt{\frac{3\,(2j_a+1)\,(2j_c+1)}{4\pi}} (-1)^{j_c - 1/2} \begin{pmatrix} j_a & 1 & j_c \\ -1/2 & 0 & 1/2 \end{pmatrix} \\ &\times (1-k) \left[\frac{1}{2} g_s - g_l (1 + \frac{k}{2}) \right], \end{aligned} \tag{45}$$

where the first factor is a radial overlap integral, and

$$k = (j_a + \frac{1}{2})(-1)^{(j_a + l_a + 1/2)} + (j_c + \frac{1}{2})(-1)^{(j_c + l_c + 1/2)}. \tag{46}$$

Here g_s and g_l are the spin and orbital gyromagnetic ratios. In our calculations, we have adopted the values suggested by the analysis found in [20], namely 0.70 of their bare values.

IV. APPLICATIONS

We have applied the theory described above to four nuclei, the odd neutron nuclei ^{157}Gd, ^{159}Dy, ^{163}Er and the odd proton case ^{157}Tb. All but ^{163}Er were studied earlier [5,8] by our methods. The results for the neutron cases, in particular, displayed $K = \frac{1}{2}$ staggering not seen in the experimental results. We added Er to the mix because it has several $K = \frac{1}{2}$ bands and has been used in the past as a classic example of the Coriolis attenuation phenomenon.

The special technical problems involved in applying our formalism have been fully documented and solved in our previous papers and will not be repeated here. As far as the size of the model space is concerned we remind the reader that for the low energy data that we are analyzing, it suffices to describe the core by its ground-state band and those few excited bands, such as beta and gamma bands (but not restricted to these) that show some quadrupole collectivity with respect to the ground band. We then choose the size of the single-particle space coupled to these bands large enough so that further enlargement does not modify the results. In this respect we can claim to have essentially exact solutions of our semi-microscopic model, though the question of whether these are close to fully self-consistent solutions of the Kerman-Klein equations remains an open question. In general we have found that three major shells centered at the valence shell suffices.

For all intents and purposes, our previous calculations contained only one free parameter, the strength κ_2 of the quadrupole-quadrupole interaction, all other parameters having been

determined by other experimental results. (A minor cavil to this assertion is that small adjustments of the spherical single-particle energies emerging from Woods-Saxon calculations were permitted for levels near the Fermi surface.) In the present work, we have a second parameter, the strength κ_1 of the magnetic interaction, to adjust.

Before displaying the results found using the full theory, we illustrate the method of fitting the data with the additional parameter by studying a simplified model involving the coupling of a single j level to the core bands of Gd and with a value of the quadrupole interaction chosen so that we have a lower $K = \frac{3}{2}$ band clearly separated from a higher $K = \frac{1}{2}$ band. We see from Fig. 1 that for fixed κ_2, the staggering is reduced and finally eliminated as we raise κ_1 from its initial value of 0. At the same time the effect on the shape of the $K = \frac{3}{2}$ band is negligible. Next, fixing the magnetic interaction at its optimum value, we see from Fig. 2 that changing the value of κ_2 shifts band-head energies. What is not clear from these figures is that in the general case, the magnetic interaction also shifts relative band-head energies. Therefore in actual calculations, we first adjust κ_2 to give the best results for other than the staggering, next find the optimum value of κ_1, and subsequently refit band heads with a new search on κ_2. This is an iterative procedure to find the best pair of coupling strengths.

We next present the results of our calculations. The odd neutron nuclei ^{157}Gd and ^{159}Dy were studied previously in [5]. We found that we could fit the energies satisfactorily except that a staggering of a $K = 1/2$ band for each nucleus that was not observed experimentally. To obtain the quality of agreement found in the previous work, it was necessary to include the β band, the γ band, and an additional $K = 0$ excited band. In Figs. 3 and 4 we show the results of the augmented calculations and compare them to those found previously. It can be seen that the addition of the magnetic dipole interaction removes the unphysical staggering found earlier without any serious damage to the rest of the fit.

The odd proton nucleus ^{157}Tb was used in a previous study of the status of the traditional core-particle model as an approximation to the KKDF model [8]. In Fig. 5 we see that in this case no exaggerated staggering was predicted by the theory without a magnetic dipole interaction. The addition of a magnetic dipole-dipole field with strength of the same order of magnitude as for the other two nuclei gives, nevertheless, a perceptibly improved overall fit to the data.

Last we have applied our theory to the case of ^{163}Er. We have studied both the negative and positive parity states of this nucleus. The results are shown in Fig. 6. In the right panel, which displays the results for the levels of odd parity, one sees that we have found a good fit to the data, which includes three $K = \frac{1}{2}$ bands. Remark that the fit is best for those two of these bands for which the data is more extensive. In the left panel, which is devoted to the positive parity levels, there is only a single $K = \frac{1}{2}$ bandhead. The lowest band, $K = \frac{5}{2}$, the only one that seems to be well-established experimentally, has features that we have failed to capture thus far.

V. J·J INTERACTION

As mentioned in the introduction we can achieve results similar to those reported above by replacing the magnetic dipole interaction used previously by a term proportional to the square of the angular momentum. In Fig. 7 we contrast the new calculation thus engendered

for ^{157}Gd (right panel) with that found in the previous section (reproduced in the left panel. We find that both interactions have a similar effect on the staggering, but that the effect on the relative bandhead energies is different. In general, fits of comparable quality are obtained. To achieve the desired staggering, the strength, κ_1' of the $\mathbf{J}\cdot\mathbf{J}$ interaction should be set at $\kappa_1 = 0.026$. We get the best fit if the strength of the quadrupole field κ_2 is altered slightly from what we had before: $\kappa_2 = 0.392$ MeV/fm^2 compared to $\kappa_2 = 0.394$ MeV/fm^2.

We compare the two interactions to understand their similarities and differences. If only matrix elements diagonal in both the core and the particle are considered, it is a well-known property of vector operators that the two interactions are equivalent. In fact this equivalence holds very well for the core, since for $K = 0$ bands, with I values differing by two, the magnetic dipole operator has no off-diagonal elements. It is true that we lose this simplification for the gamma band, but the physics we are discussing is dominated by the ground band. When we turn, however, to the single-particle space, off-diagonal matrix elements of the magnetic moment operator can play an important role, as in generating magnetic dipole transitions, and are thus the agent responsible for the slightly different results found for the two interactions considered.

VI. DISCUSSION

The aim of this paper has been to present a reasonably convincing case, both on the basis of à priori arguments and by illustrative calculations, that we have uncovered the origin of the phenomena referred to as attenuation of the Coriolis coupling. By basing our considerations on a theoretical framework of great generality, we were able to rule out a large subset of explanations offered previously, since we obtained evidence of the attenuation phenomenon even when the physics associated with these explanations was naturally incorporated in our calculations. The only possibility remaining to us was to think about adding some term to the pairing and quadrupole interactions that define the Hamiltonian used in our previous work. Because of the nature of the effect sought, the most obvious choice is a magnetic dipole interaction. There are two such interactions that can be fitted easily into our semi-microscopic theory, the scalar product of the magnetic moment operator with itself and the square of the angular momentum operator. Calculations done with the former on four different nuclei showed that the excessive staggering encountered in previous calculations, but absent experimentally, could be made to disappear with a dipole interaction strength that remains constant within 10% for the nuclei studied. For one of the nuclei, ^{157}Tb, where no excessive staggering was calculated previously, we found a slight improvement to the overall fit with the same interaction strength.

For one case, that of ^{157}Gd, we compared the results found for the two types of interaction mentioned above and found quite similar, but not identical, results. This suggests that we have identified the multipolarity of the additional interaction necessary to explain the quenching effect, but not its detailed form.

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TABLES

nucleus	g_1 [μ_ν]	κ_1 [MeV/μ_ν^2]	κ_2 [MeV/fm^2]
^{157} Gd	0.371	0.045	0.394
^{159} Dy	0.345	0.043	0.368
^{157} Tb	0.362	0.052	0.420
^{163} Er (-)	0.353	0.04	0.366
^{163} Er (+)	0.353	0.04	0.396

TABLE I. Parameters used in the calculations reported in this work. The second column lists the gyromagnetic ratios of the core nuclei, the third column the values of the magnetic dipole interaction strenght, and the last the values for the quadrupole interaction strenght.

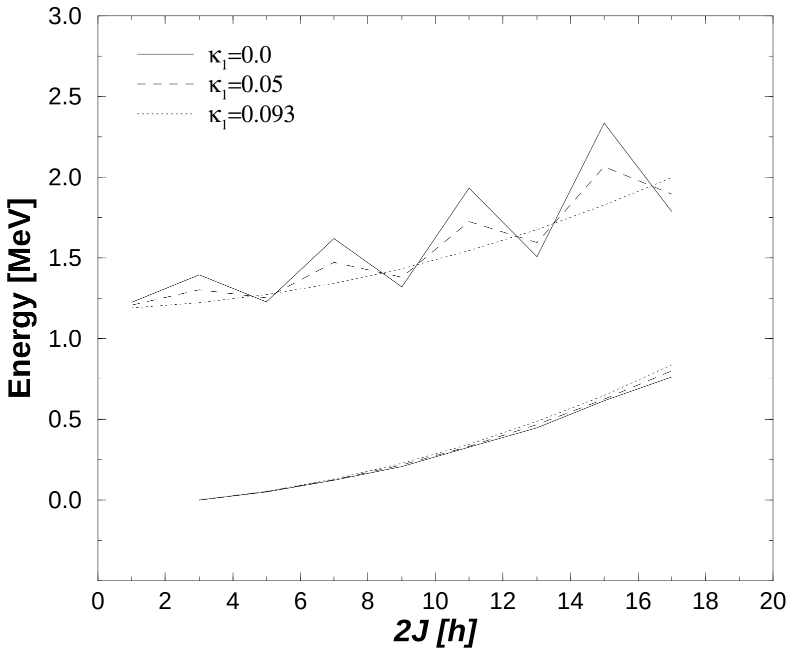

FIG. 1. A model calculation to illustrate the effect of the magnetic dipole-dipole interaction on staggering. A single-particle space with a single j value is coupled to the ground state bands of the even neighbors of ^{157}Gd. The strength of the quadrupole force is fixed at $\kappa_2 = 0.564 \mathrm{MeV/fm}^2$. The strength of the magnetic dipole force is varied to demonstrate the effect of this interaction.

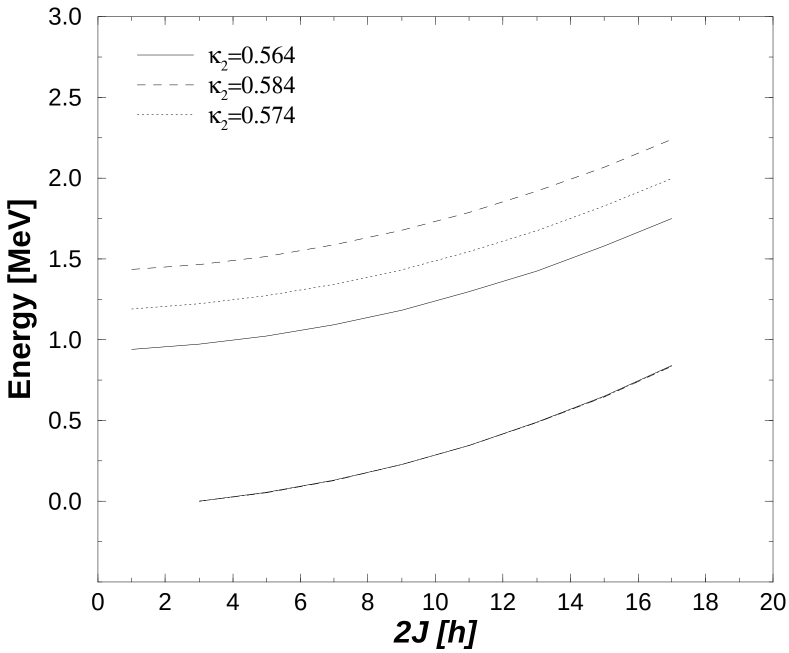

FIG. 2. The purpose of this figure is to demonstrate that the strength of the quadrupole field does not influence the staggering. The model is the same as in Fig 1. The value, κ_1, of the magnetic interaction is set at the value that minimizes the staggering of the $K = 1/2$ band. Then the effect of varying κ_2 is studied.

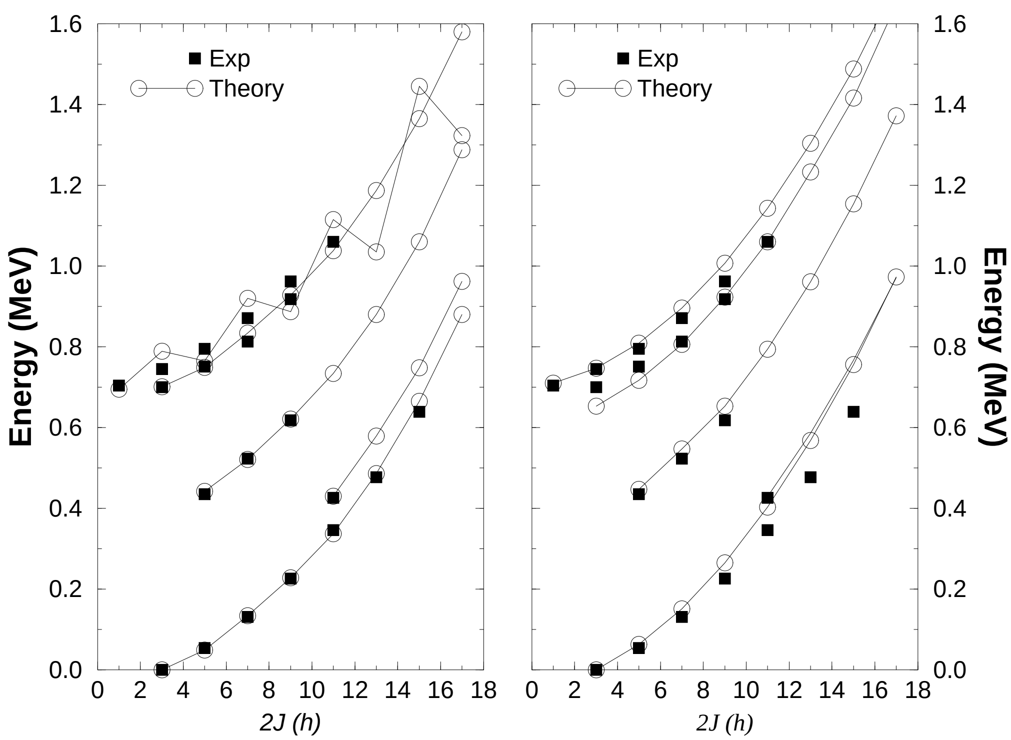

FIG. 3. Negative parity energy levels of ^{157}Gd. The calculation includes single particle states for three major shells and excited bands of the core. On the left is the previous calculation, with the strength of the quadrupole force set at 0.380 MeV/fm^2 and $\kappa_1 = 0.0$ MeV/μ_ν^2. On the right is the improved calculation with $\kappa_2 = 0.394$ MeV/fm^2 and $\kappa_1 = 0.045$ MeV/μ_ν^2.

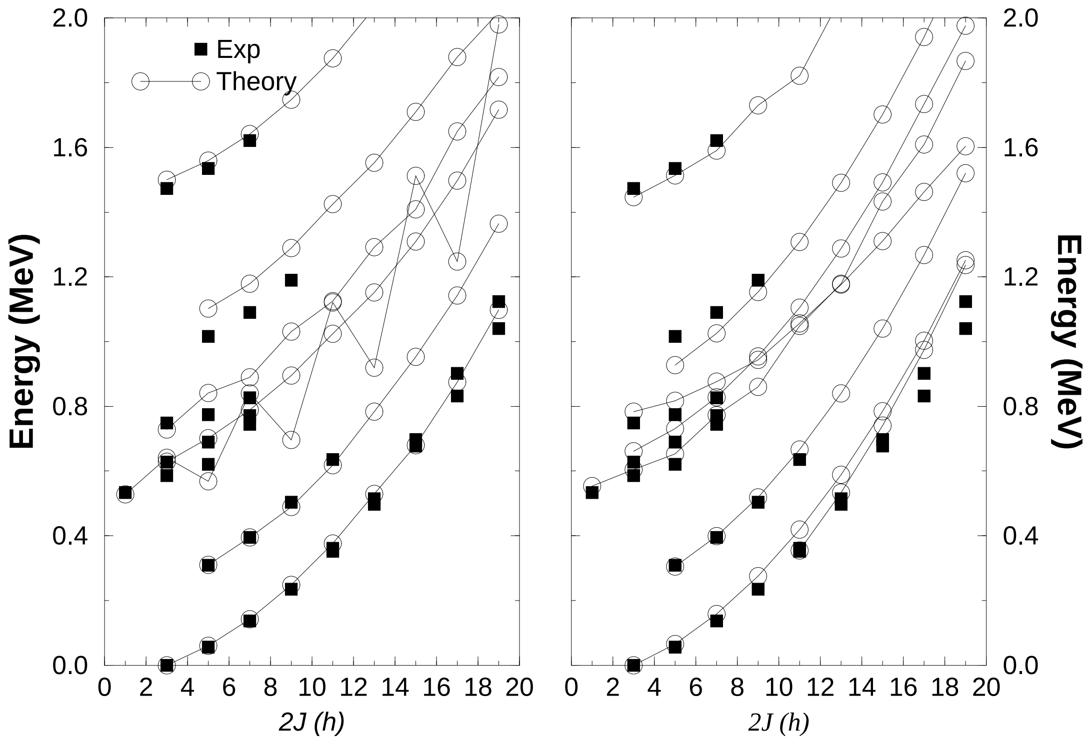

FIG. 4. Negative parity energy levels of ^{159}Dy. The calculation includes single particle states for three major shells and excited bands of the core. On the left is the previous result with the strength of the quadrupole force set at 0.383 MeV/fm^2 and $\kappa_1 = 0.0$ MeV/μ_ν^2. On the right is the improved calculation with $\kappa_2 = 0.368$ MeV/fm^2 and $\kappa_1 = 0.043$ MeV/μ_ν^2.

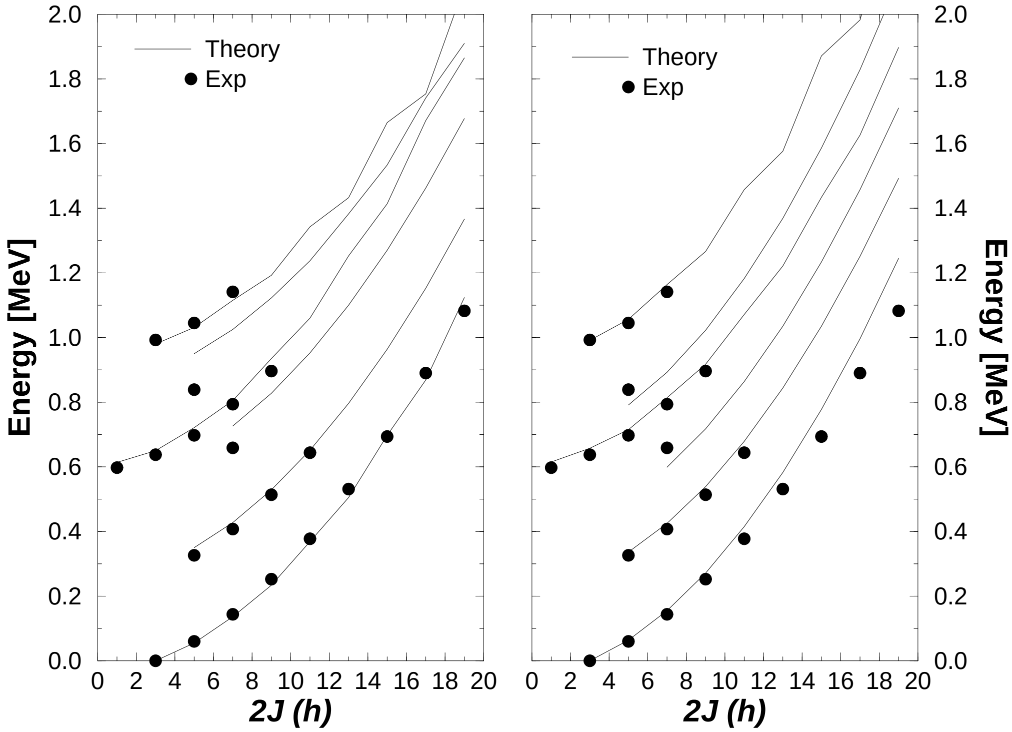

FIG. 5. Positive parity energy levels of ^{157}Tb. The calculation includes single particle states for three major shells and excited bands of the core. On the left is shown the previous result where the strength of the quadrupole force is 0.428 MeV/fm^2 and $\kappa_1 = 0.0$ MeV/μ_ν^2. On the right is the improved calculation with $\kappa_2 = 0.420$ MeV/fm^2 and $\kappa_1 = -0.052$ MeV/μ_ν^2.

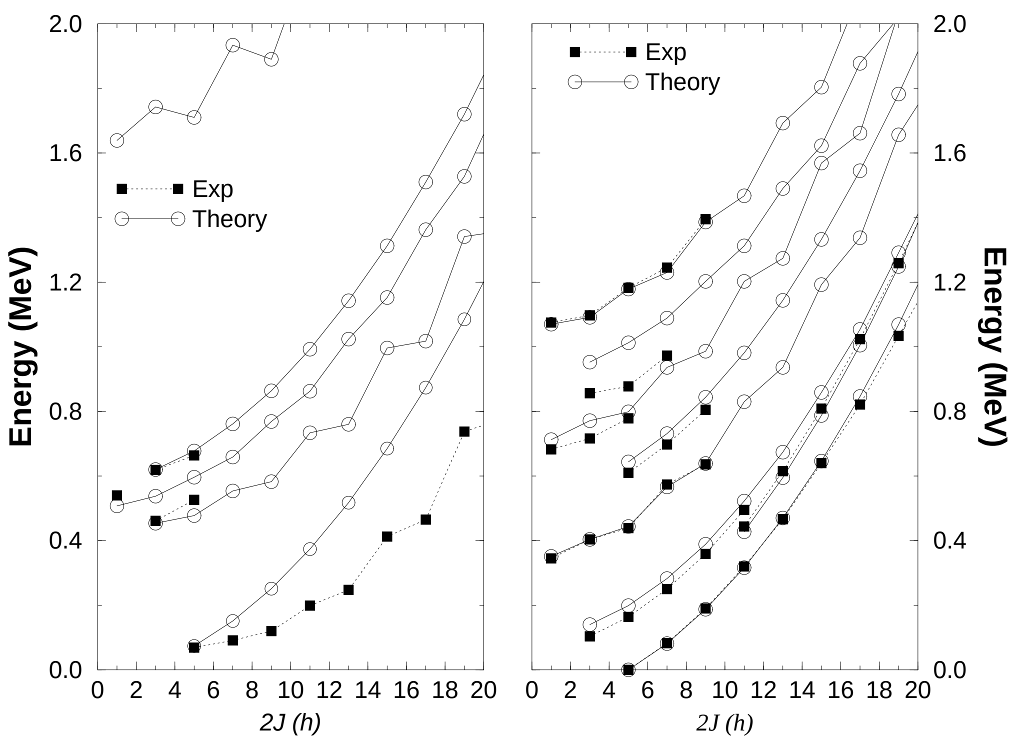

FIG. 6. Energy levels of ^{163}Er. Single particle states for three major shells and excited bands of the core are included. On the left are the positive parity states and on the right the negative parity states. For the positive parity states the strength of the quadrupole force is 0.413 MeV/fm^2 when $\kappa_1 = 0.0$ MeV/μ_ν^2 (not shown) and $\kappa_2 = 0.399$ MeV/fm^2 when $\kappa_1 = -0.040$ MeV/μ_ν^2. For the negative parity states the strength of the quadrupole force is 0.375 MeV/fm^2 when $\kappa_1 = 0.0$ MeV/μ_ν^2 (not shown) and $\kappa_2 = 0.366$ MeV/fm^2 when $\kappa_1 = -0.040$ MeV/μ_ν^2.

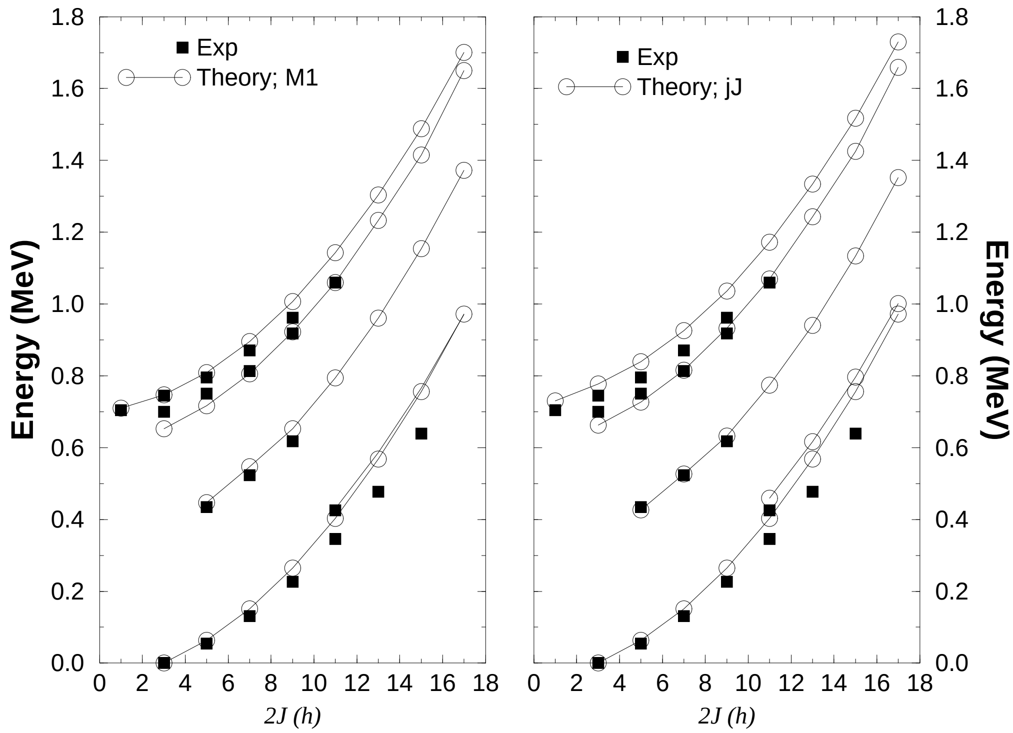

FIG. 7. Comparison of two magnetic interactions. On the left is the same fit as in Fig. 3 using the magnetic dipole operator and on the right the fit using the angular momentum form. For the calculations in the left panel, we have $\kappa_2 = 0.394$ MeV/fm^2 with $\kappa_1 = 0.045$ MeV/μ_ν^2;. for the calculation on the right, $\kappa_2 = 0.392$ MeV/fm^2 and $\kappa_1' = 0.026$ MeV/μ_ν^2.